Period 1, Jan 15, 2025

8. Given $g(x) = ln(x + 1)$, use the equation of the tangent line of $g(x)$ at $x = 0$ to find the approximation of $ln(1.1)$.

$g(0) = \ln(0+1) = \ln(1) = 0$

Point $(0,0)$

$g'(x) = \frac{1}{x+1} \cdot (1) = \frac{1}{x+1}$

$g'(0) = \frac{1}{0+1} = 1$ $m=1$ Poin $(0,0)$

$y = x$ $\ln(1.1)$ $x = 0.1$

$x = 0.1$

$y = 0.1$

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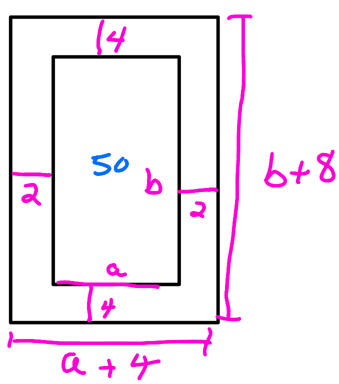
4. A poster is to contain 50 square inches of printed matter with margins of 4 inches at both the top and the bottom and 2 inches at each side. Find the dimensions that will minimize the total area of the poster.



$a \cdot b = 50 \Rightarrow \boxed{a = \frac{50}{b}}$

$Area = (a+4)(b+8)$ $b=10$

$A = (\frac{50}{b}+4)(b+8)$ $A = \frac{50}{b}$

$A = 50 + \frac{400}{b} + 4b + 32$

$A = 82 + 400b^{-1} + 4b$

$\frac{dA}{db} = 0 + 400 \cdot -1 \cdot b^{-2} + 4 = 0$

$\frac{-400}{b^2} = -4$

$b = \pm 10$ b is a distance $b = 10$

Max/min

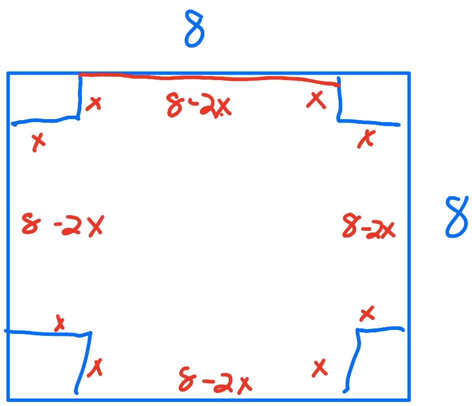
$\frac{dA}{db} = -400b^{-2} + 4$

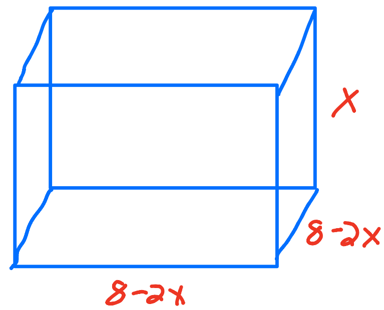
$\frac{d^2A}{db^2} = 800b^{-3} + 0$

always + if b is positive

concave up/min

5. A tinsmith wishes to make an open top box from a square piece of tin which measures 8 in. by 8 in. To accomplish this task, he proposes to cut equal square pieces from each corner of the tin and fold up the tin to form sides. Determine the length of the sides of the squares to be cut from the corners so that the box will have the greatest possible volume.





$3x^2-16x+16=0$

$3\cdot16=48$

$12+4=16$

$3x^2-12x-4x+16$

$3x(x-4)-4(x-4)$

$(x-4)(3x-4)=0$

$V=x(8-2x)(8-2x)$

$V=x(64-32x+4x^2)$

$V=4x^3-32x^2+64x$

$\frac{dV}{dx}=12x^2-64x+64=4(3x^2-16x+16)$

2. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

$a\cdot b=192$

$a=\frac{192}{b}$

$b=8$

$a=\frac{192}{8}=24$

$a+3b=min$

$a+3b=m$

$\frac{192}{b}+3b$

$192\cdot b^{-1}+3b=min$

$-192b^{-2}+3=0$

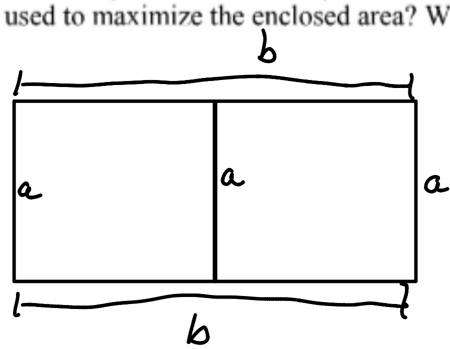
$\frac{-192}{b^2}=-3$

$\frac{-192}{-3}=\frac{-3b^2}{-3}$

$64=b^2$

$\pm8=b$

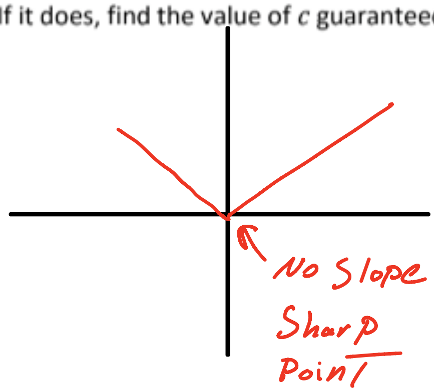
3. A farmer has 360 feet of fencing to enclose two adjacent and identical rectangular corrals. What dimensions should be used to maximize the enclosed area? What is the maximum area? Include units in your answer.



$3a+2b=360$

$A=ab$

6. Determine if the Mean Value Theorem applies to the function $f(x) = |x|$ over the interval $[-2,2]$. If it does not, explain why. If it does, find the value of c guaranteed by the theorem.



7. A small balloon is released at a point 150 feet away from an observer who is on level ground. If the balloon goes straight up at the rate of 8 ft/sec, how fast is the distance from the observer to the balloon increasing when the balloon is 50 ft. high?

$h = 50$ $\qquad \frac{dh}{dT} = 8 \text{ FT/sec}$ $\qquad \frac{dL}{dT} = ?$

$h^2 + 150^2 = L^2$

$2h\frac{dh}{dT} = 2L\frac{dL}{dT}$

$50^2 + 150^2 = L^2$

$2500 + 22500 = L^2$

$25000 = L^2$

$\sqrt{2500 \cdot 10} = L$

$50\sqrt{10} = L$

$y = 4x^5 + 7x^2 - 12x + 14$

$\frac{dy}{dx} = 20x^4 + 14x - 12 + 0$

$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = (20x^4 + 14x - 12) \cdot dx$

$\int dy = \int (20x^4 + 14x - 12)\,dx$

$y = 20 \cdot \frac{1}{5} \cdot x^{4+1=5} + 14 \cdot \frac{1}{2} \cdot x^{1+1=2} - 12 \cdot \frac{1}{1} \cdot x^{0+1=1} + C$

$y = 4x^5 + 7x^2 - 12x + C$

$\int (5x^7 + 12x^{14} - 17x + 4)\,dx$

$\frac{5}{8}x^8 + \frac{4}{5}x^{15} - \frac{17}{2}x^2 + 4x + C$

$\int (3\sin x + 2\sec^2 x)\,dx$

$-3\cos x + 2 \cdot \tan x + C$

$\int \frac{1}{x}\,dx = \ln|x|$

$\int \left(\frac{7x^4 + 12 - 16x^3 + 4x}{x^2} \right) dx$

$\int \left(\frac{7x^4}{x^2} + \frac{12}{x^2} - \frac{16x^3}{x^2} + \frac{4x}{x^2} \right) dx = \int \left(\underbrace{7x^2 + 12x^{-2} - 16x + \frac{4}{x}} \right) dx$

$7 \cdot \frac{1}{3} \cdot x^{2+1=3} + 12 \cdot \frac{1}{-1} x^{-2+1=-1} - 16 \cdot \frac{1}{2} \cdot x^{1+1=2} + 4\ln|x| + C$

$$y=\frac{7}{3}x^3-12x^{-1}-8x^2+4\ln x+c$$

$$\frac{dy}{dx}=\frac{7}{\cancel{3}}\cancel{3}\cdot x^{3-1=2}-12\cdot -1\cdot x^{-1-1=-2}-8\cdot 2\cdot x^{2-1=1}+4\cdot\frac{1}{x}+0$$

$$\frac{dy}{dx}=7x^2+12x^{-2}-16x+\frac{4}{x}$$

$$\int 4x^{-1}dx=4\cdot\frac{1}{0}\cdot x^{-1+1=0}+c=\phi$$

$$\frac{d}{dx}[\ln x]=\frac{1}{x}=x^{-1}$$

$$\int x^{-1}dx=\int\frac{1}{x}dx=\ln|x|+c$$

$$y=x^3+4x^2-2x+7$$ when $x=-1$ $y=12$

$$\frac{dy}{dx}=3x^2+8x-2$$

$$\int(3x^2+8x-2)\,dx$$ when $x=-1$ $y=12$ Find original Function

$$3\cdot\frac{1}{3}\cdot x^{2+1=3}+8\cdot\frac{1}{2}\cdot x^{1+1=2}-2\cdot\frac{1}{1}\cdot x^{0+1=1}+c$$

$$y=x^3+4x^2-2x+c$$

$$12=(-1)^3+4(-1)^2-2(-1)+c \Rightarrow \boxed{y=x^3+4x^2-2x+7}$$

$$12=-1+4+2+c$$

$$12=5+c$$

$$7=c$$

$S(T) = \text{Position}$

$S'(t) = V(T) = \text{Velocity}$

$S''(t) = V'(T) = a(t) = \text{accel}$

$\int v(T)\,dT = S(T)$

$\int a(T)\,dt = V(T)$

$F''(x) = \frac{d^2y}{dx^2} = \frac{2}{5}x^3 + 4x^2 - 5$

$F'(5) = -3$

$F(1) = 7$

$\int \left(\frac{2}{5}x^3 + 4x^2 - 5\right)dx$

$F'(x) = \frac{2}{5} \cdot \frac{1}{4} \cdot x^{3+1=4} + 4 \cdot \frac{1}{3} \cdot x^{2+1=3} - 5x + C_1$

$F'(x) = \frac{x^4}{10} + \frac{4}{3}x^3 - 5x + C_1$

$-3 = \frac{5^4}{10} + \frac{4}{3}(5)^3 - 5(5) + C_1$

$-3 = 62.5 + \frac{500}{3} - 25 + c_1$

$-3 = 37.5 + 166\frac{2}{3} + C_1$

$-3 = 37\frac{1}{2} + 166\frac{2}{3} + C_1$

$-3 = 203\frac{7}{6} + C_1$

$-3 = 204\frac{1}{6} + C_1$

$-207\frac{1}{6} = C_1$

$F'(x) = \frac{x^4}{10} + \frac{4}{3}x^3 - 5x - 207\frac{1}{6}$

$F(x) = \int F'(x)\,dx$

$F(x) = \frac{1}{10} \cdot \frac{1}{5} \cdot x^{4+1=5} + \frac{4}{3} \cdot \frac{1}{4} \cdot x^{3+1=4} - \frac{5}{2}x^2 - 207\frac{1}{6}x$

$F(x) = \frac{x^5}{50} + \frac{1}{3}x^4 - \frac{5}{2}x^2 - 207\frac{1}{6}x + 216\frac{47}{150}$ $+C_2$

$F(x) = \frac{x^5}{50} + \frac{1}{3}x^4 - \frac{5}{2}x^2 - 207\frac{1}{6}x + C_2$

$7 = \frac{1^5}{50} + \frac{1}{3}(1)^4 - \frac{5}{2}(1)^2 - 207\frac{1}{6}(1) + C_2$

$7 = \frac{1}{50} + \frac{1}{3} - \frac{5}{2} - 207\frac{1}{6} + C_2$

$\frac{3}{150} + \frac{50}{150} - \frac{375}{150} - 207\frac{1}{6} + C_2$

$7 = \frac{-322}{150} - 207\frac{25}{150} + C_2$

$7 = -2\frac{22}{150} - 207\frac{25}{150} + C_2$

$7 = -209\frac{47}{150} + C_2$

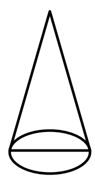
$C_2 = 216\frac{47}{150}$

$y=\sqrt[3]{x^2+1}=(x^2+1)^{\frac{1}{3}}$ $\qquad \frac{dx}{dT}=4\ \text{cm/min}$ $\qquad (0,1)$ $\quad x=0$

$\frac{dy}{dT}=\frac{1}{3}(x^2+1)^{-\frac{2}{3}}\cdot 2x$

$\frac{dy}{dT}=\frac{2x}{3\sqrt[3]{x^2+1}}\frac{dx}{dT}=\frac{2\cdot 0}{3\sqrt[3]{0^2+1}}\ \frac{dx}{dx}$

$\frac{dy}{dT}=0$



$\frac{dV}{dT}=11\ \text{FT}^3/\text{min}$

$r=\boxed{\frac{1}{3}h}$

$\frac{dh}{dT}=?$

when $h=5$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\cdot\pi\left(\frac{h}{3}\right)^2\cdot h = V = \frac{\pi}{3}\cdot\frac{h^2}{9}\cdot h$

$V=\frac{\pi}{27}h^3$

$\frac{dV}{dT}=\frac{\pi}{27}\cdot 3h^2\frac{dh}{dT}$

$11=\frac{\pi}{27}\cdot 3(5)^2\frac{dh}{dT}=\frac{\pi\cdot 25}{9}\frac{dh}{dT}$

$\frac{9}{25\pi}\cdot 11=\cancel{\frac{25\pi}{9}}\cdot\frac{dh}{dT}\cdot\cancel{\frac{9}{25\pi}}$

$\frac{dh}{dT}=\frac{99}{25\pi}$

$\frac{dV}{dT} = 12 \ FT^3/min$ $\qquad \boxed{r = \frac{1}{3}h}$

when $h = 5$

$V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{\pi}{27} h^3$

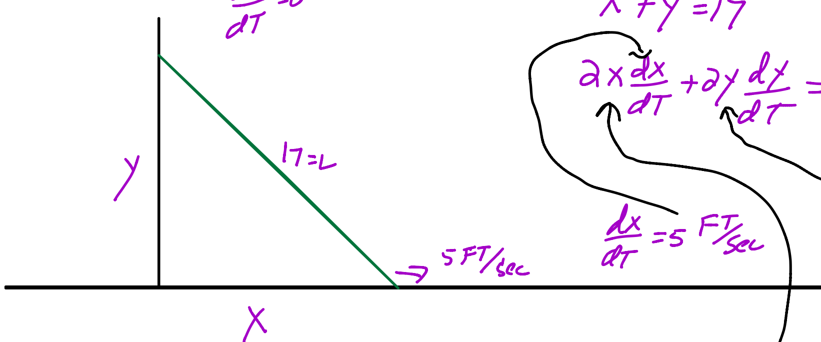
$\frac{dV}{dT} = \frac{\pi}{9} h^2 \frac{dh}{dT}$

$12 = \frac{\pi}{9}(5)^2 \frac{dh}{dT}$

$\frac{9}{25\pi} \cdot 12 = \frac{\cancel{25\pi}}{\cancel{9}} \frac{dh}{dT} \cdot \frac{\cancel{9}}{\cancel{25\pi}}$

$\frac{108}{25\pi} = \frac{dh}{dT}$

$\frac{dL}{dT} = 0$



$x^2 + y^2 = 17^2$

$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$

$\frac{dy}{dT} = ?$

when $y = 15$

$\frac{dx}{dT} = 5 \ FT/sec$

$x^2 + 15^2 = 17^2$

$x^2 + 225 = 289$

$x^2 = 64$

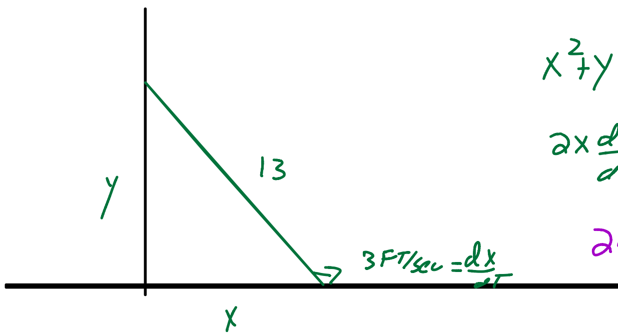
$x = 8$

$2(8) \cdot 5\frac{FT}{sec} + 2(15) \cdot \frac{dy}{dT} = 0$

$80 + 30\frac{dy}{dT} = 0$

$30\frac{dy}{dT} = -80$

$\frac{dy}{dT} = -\frac{8}{3} \ FT/sec$



$$x^2 + y^2 = 13^2$$

$$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$$

$$2 \cdot 5 \cdot 3 + 2 \cdot 12 \cdot \frac{dy}{dT} = 0$$

$$30 + 24\frac{dy}{dT} = 0$$

$$24\frac{dy}{dT} = -30$$

$$\frac{dy}{dT} = \frac{-30}{24} = \frac{-5}{4} \text{ FT/sec}$$

$$y = 12$$

$$12^2 + x^2 = 13^2$$

$$144 + x^2 = 169$$

$$x^2 = 25$$

$$x = 5$$